

# Contribution of Colour-singlet Process $\Upsilon \rightarrow J/\Psi + c\bar{c}g$ to $\Upsilon \rightarrow J/\Psi + X$

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## Abstract

We show that the colour-singlet process  $\Upsilon \rightarrow J/\Psi + c\bar{c}g$  significantly contributes to the inclusive process  $\Upsilon \rightarrow J/\Psi + X$ . We calculate the partial width and the momentum distribution of the produced  $J/\Psi$  in this channel. The obtained width is comparable to the experimental data. The momentum distribution is fairly soft, which is in contrast to the results from the colour-octet processes discussed earlier in literature but is consistent with the CLEO measurements. Further experiments to check this contribution are suggested.

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Heavy quarkonium decay and production are traditional arena for QCD. The descriptions [1] of such processes are nearly standard as a factorization procedure: The annihilation of  $b\bar{b}$  or  $c\bar{c}$  to gluon(s) or its creation from gluon(s) can be calculated via perturbative QCD, since the heavy quark mass sets a hard scale for corresponding processes. The bound state of the heavy quarkonium contains non-perturbative effects. This part is ‘factorized out’ and is described by QCD-inspired models [2]. Such a procedure is successful in describing many experimental data. However, recently a large discrepancy for the prompt production of  $J/\Psi$  and  $\Psi'$  has been observed in Fermilab Tevatron between the data [3] and the colour-singlet model prediction. This lead to a re-inspection of the description of heavy quarkonium decay and production (for a review, see [4]). It was found that the ‘factorization’ in colour-singlet model had never been proved. And the colour-octet mechanism which is based on the NRQCD factorization scheme (see, e.g.[5]) was proposed as a complement to the colour-singlet mechanism to explain the above mentioned discrepancy. It has been shown that the colour-octet mechanism plays an important rôle in many processes. For example, it has been argued that two processes  $\Upsilon \rightarrow J/\Psi + gg$  [6] (see Fig.1(a)) and  $\Upsilon \rightarrow J/\Psi + g$  [7] (see Fig.1(b)) where the colour-octet mechanism plays the key rôle provide the leading contribution to the inclusive process  $\Upsilon \rightarrow J/\Psi + X$ . Experiments by the CLEO collaboration reported a branching ratio of  $(1.1 \pm 0.4 \pm 0.2) \times 10^{-3}$  and quite a soft  $J/\Psi$  momentum spectrum (see dots in Fig.3) [8]. The calculated branching ratios of these two colour-octet processes are indeed comparable to the data. However, the  $J/\Psi$  momentum spectra of both processes peak at the largest value  $\sim 4.2GeV/c$ , which is in strong contradiction to the CLEO data. (For  $\Upsilon \rightarrow J/\Psi + gg$ , see the dotted line in Fig.3; for  $\Upsilon \rightarrow J/\Psi + g$ , the  $J/\Psi$  momentum spectrum is a  $\delta$ -function with the pole at  $\sim 4.2GeV/c$ .)

In this note, we show that a colour-singlet process,  $\Upsilon \rightarrow J/\Psi + c\bar{c}g$  (see Fig.2), also significantly contributes to the inclusive  $J/\Psi$  production in  $\Upsilon$  decay, and that the  $J/\Psi$  momentum spectrum from this channel has to be soft, which is consistent with the CLEO data. The latter point can be seen from the following simple kinematic analysis. We recall that angular momentum conservation and charge conjugation invariance demand that  $\Upsilon$  decay mainly through three (real or virtual) gluons. It is clear that  $\Upsilon$  can decay via one real and two virtual gluons, and both of the virtual gluons split into colour-octet  $c\bar{c}$  (see Fig.2). In this case, a  $c$  and a  $\bar{c}$  originated from different gluons can be in the colour-singlet, and they can form a  $J/\Psi$  when they are in the proper angular momentum state and their invariant mass is near  $J/\Psi$  mass shell. Generally, the remaining  $c\bar{c}$  created in this process hadronizes into open charm particles, so the final particle system excluding  $J/\Psi$  has a minimum invariant mass  $2M_{D^0}$ . This implies that the  $J/\Psi$  momentum has a maximum of 3.3GeV/c in this channel. From the Feynman diagram, we see also no indication that the  $J/\Psi$  momentum spectrum peaks at this largest value, so it should be soft. The exact shape depends on the details of the dynamics which is discussed later in this paper. We also note that if only one instead of two  $c\bar{c}$  is created in the 3-gluon decay channel, which to the lowest orders in  $\alpha_s$  correspond to the above mentioned two colour-octet processes [6, 7], the  $J/\Psi$  momentum spectrum is very hard and in strong contrast to the CLEO data, as cited at the end of last paragraph. Hence we conclude that in the 3-gluon decay channel and to the lowest orders in  $\alpha_s$ , the process in Fig.2 is the only case where the  $J/\Psi$  momentum spectrum is soft.

The subsequent question is then: how large is the branching ratio of this colour-singlet process? At first sight, one may expect that it is much smaller than that of the colour-octet process  $\Upsilon \rightarrow J/\psi + gg$  discussed in [6], since in

the perturbative (short-distance) phase the colour-singlet process has one more  $\alpha_s$  and one more virtual gluon propagator, both of which give more suppression. But a careful inspection tells us that it can be of the same order of magnitude as that from the colour-octet process  $\Upsilon \rightarrow J/\psi + gg$ , thus comparable to the data [8]. The reason is as follows. It is true that in the perturbative phase the colour-singlet process we consider receives more suppression than the colour-octet process [6]. But in the non-perturbative (long-distance) phase, where the colour-singlet/octet  $c\bar{c}$  form the  $J/\psi$  resonance with the probability described by the colour-singlet/octet matrix elements [5], the much larger colour-singlet matrix element provides an enhancement. Using the value of [6], the colour-singlet matrix element  $\langle O_1^{J/\Psi}(^3S_1) \rangle$  is about 52 times as large as the colour-octet one  $\langle O_8^{J/\Psi}(^3S_1) \rangle$ . If we use the more updated value in [9], the ratio  $\frac{\langle O_1^{J/\Psi}(^3S_1) \rangle}{\langle O_8^{J/\Psi}(^3S_1) \rangle}$  can even be 210 to 360. This enhancement can compensate the perturbative suppression. So we expect a non-negligible partial width of the colour-singlet process  $\Upsilon \rightarrow c\bar{c}(^3S_1, 1) + c\bar{c}g \rightarrow J/\Psi + c\bar{c}g$ , where we use  $(^3S_1, 1)$  to denote the quantum numbers of angular momentum and colour for  $c\bar{c}$ . Furthermore, if the invariant mass of the colour-singlet pair  $c\bar{c}(^3S_1)$  is near the mass shell of  $\Psi(2S)$ , it can also form a  $\Psi(2S)$  resonance, which decays into  $J/\Psi$  with a branching ratio of 54.2% [10]. This indirect contribution  $\Upsilon \rightarrow \Psi(2S) + c\bar{c}g \rightarrow J/\Psi + X$  provides another source of the inclusive  $J/\Psi$  production in  $\Upsilon$  decay.

Encouraged by these qualitative analyses we calculate the partial width of  $\Upsilon$  and momentum distribution of  $J/\Psi(\Psi(2S))$  for the direct process shown in Fig.2 and the process  $\Upsilon \rightarrow c\bar{c}(^3S_1, 1) + c\bar{c}g \rightarrow \Psi(2S) + c\bar{c}g$ . Since both of them are colour-singlet processes and the charm quark is heavy, the treatment for the bound states can be a conventional wave-function approach where the relative momentum between  $c$  and  $\bar{c}$  is vanishing, namely same as the case of positronium

(see, e.g. [11]). Similar to [11], the differential width of the direct process (see Fig.2) can be formulated as

$$\frac{d\Gamma}{dR} = \frac{|B_{J/\Psi} B_{\Upsilon} < c\bar{c}(^3S_1, 1) c\bar{c}g | \mathcal{S} | b\bar{b}(^3S_1, 1) >|^2}{T} \quad (1)$$

where  $dR$  is the 8-dimensional phase space volume element for  $J/\Psi$  and  $c, \bar{c}, g$ ;  $\mathcal{S}$  is the S-Matrix;  $B_{J/\Psi}$  and  $B_{\Upsilon}$  are related to the origin values of the wave functions of  $J/\Psi$  and  $\Upsilon$ :

$$B_{J/\Psi} = \frac{\Psi_{J/\Psi}^*(0)}{\sqrt{m_c}}, \quad (2)$$

$$B_{\Upsilon} = \frac{\Psi_{\Upsilon}(0)}{\sqrt{V}2m_b}. \quad (3)$$

For convenience, we normalize all particle states to  $2EV$  (where  $E$  is the particle's energy and  $V$  is the volume of the total space) except for  $\Upsilon$ , which is normalized to 1. This is why the coefficients of the wave functions in (2) and (3) are different. In (1) the sum over all spin states for final particles and average of the 3 spin states for  $\Upsilon$  are not explicitly shown and the 'time'  $T$  is  $2\pi\delta(0)$ . The corresponding expression for the differential width of the process  $\Upsilon \rightarrow c\bar{c}(^3S_1, 1) + c\bar{c}g \rightarrow \Psi(2S) + c\bar{c}g$  is the same. The only differences lie in the phase space and the wave function at the origin, where the corresponding quantities for  $\Psi(2S)$  should be used to replace those of  $J/\Psi$ .

For the perturbative sub-process  $b\bar{b}(^3S_1, 1) \rightarrow g^*g^*g \rightarrow c\bar{c}c\bar{c}g$ , we consider, as usual, only the six lowest-order Feynman diagrams at the tree-level (see Fig.2). The treatment of the infrared behaviour of the real gluons in these diagrams is simple. It is much similar to that in the  $\Upsilon \rightarrow 3g$  process (see, e.g. [5]). For the two diagrams where the real gluon vertex lies between the two virtual gluons, there is no infrared singularity due to 'controlling momentum' [5]. For the rest four diagrams, the infrared gluon in each diagram leads to an eikonal vertex and an eikonal quark propagator [5]. These four diagrams form two pairs. In each pair,

the only difference between the two diagrams is the position of the real gluon vertex. The eikonal vertices in them have opposite signs, so the infrared singularities of the two diagrams cancel with each other. Summing over all the six diagrams, we obtain the infrared safe matrix element  $\langle c\bar{c}({}^3S_1, 1)c\bar{c}g|\mathcal{S}|b\bar{b}({}^3S_1, 1)\rangle$ , which is needed in calculating  $\frac{d\Gamma}{dR}$  from Equation(1).

To get the numerical results, we need to know the value of  $\alpha_s$  and those of the  $\Psi_{J/\Psi}(0)$  and  $\Psi_{\Upsilon}(0)$  (see Equation(2),(3)). In order to compare the results with those of the colour-octet processes [6, 7], we use the same values for these parameters. Especially, the scale of  $\alpha_s$  is  $M_{J/\Psi}$ . The square of the radial wave function at the origin for  $\Psi(2S)$  have been calculated using different potentials [12], Consistent with these calculations, we use  $\frac{|\mathcal{R}_{\Psi(2S)}(0)|^2}{|\mathcal{R}_{\Psi(1S)}(0)|^2} \sim 1/2$  approximately, where  $\mathcal{R}$  represents the radial wave function. The quark masses are taken as  $m_c = \frac{M_{J/\Psi}}{2}$  and  $m_b = \frac{M_{\Upsilon}}{2}$ . The partial width of the direct production process  $\Upsilon \rightarrow c\bar{c}({}^3S_1, 1) + c\bar{c}g \rightarrow J/\Psi + c\bar{c}g$  is calculated to be 28.4eV. Normalized by the full width of  $\Upsilon$  [10], we obtain the corresponding branching ratio of about  $0.54 \times 10^{-3}$ . The indirect contribution to the branching ratio from  $\Psi(2S)$  is small  $\sim 0.05 \times 10^{-3}$ . So the branching ratio from these two sources is  $0.59 \times 10^{-3}$ , which is consistent with the CLEO result within the error.

If we take sum of the branching ratio obtained above for the colour-singlet process we consider, and those for the colour-octet processes discussed in [6, 7], we obtain the total branching ratio of  $1.21 \times 10^{-3}$ . This result is still in agreement with CLEO data (but is larger than the ARGUS upper limit). Because the value of the colour-singlet matrix elements can be extracted from the pure leptonic decays of  $\Upsilon$  and  $J/\Psi$  etc., which are electromagnetic process and where only the colour-singlet mechanism works, the uncertainty is rather small. This is in contrast to the case of extracting the value of the colour-octet matrix elements.

The latter can only be extracted from  $J/\Psi$  production experiments. It is therefore very sensitive to whether all the colour-singlet processes have been considered. Furthermore, the perturbative part is sensitive to the value of  $\alpha_s$ . It is unclear whether  $\alpha_s(M_{J/\Psi})$  or  $\alpha_s(M_\Upsilon)$  should be used [7]. If  $\alpha_s(M_\Upsilon)$  instead of  $\alpha_s(M_{J/\Psi})$  is used in both the colour-singlet process we considered here and the colour-octet processes considered in [6, 7], the branching ratio of the colour-singlet process decreases to  $0.12 \times 10^{-3}$  which is still of the same order of magnitude as that of the colour-octet processes. But the theoretical total width is smaller than the CLEO data. It is now impossible to judge which mechanism is more important if we only look at their contributions to the branching ratio.

In contrast to the branching ratio, the shape of the calculated  $J/\Psi$  momentum spectrum does not depend on the values of  $\alpha_s$  and the non-perturbative matrix elements. It is not influenced by the uncertainties coming from these parameters. Hence it can act as a ‘label’ for a given decay channel. The  $J/\Psi$  momentum distribution for the direct colour-singlet process (see Fig.2) is shown in Fig.3 by the solid line. It has a peak near  $2.2\text{GeV}/c$ . The shape is similar to the CLEO data. About 90% of the total events reside in the range smaller than  $3\text{ GeV}/c$ , which is also consistent with the CLEO data. The momentum spectrum of  $\Psi(2S)$  is shown by the dashed line, and the magnitudes of the differential width have been multiplied by the branching ratio of  $\Psi(2S) \rightarrow J/\Psi + X'$ . In the process  $\Psi(2S) \rightarrow J/\Psi + X'$ , since  $M_{\Psi(2S)} - M_{J/\Psi} \ll M_{J/\Psi}$ ,  $J/\Psi$  carries most of the energy of  $\Psi(2S)$ . Hence the shape of the corresponding  $J/\Psi$  momentum spectrum is approximately the same as the  $\Psi(2S)$ . We see from Fig.3 that the spectrum shape in the indirect process is similar to that in the direct process. For comparison, we also show in Fig.3 the momentum spectrum of the colour-octet process (see

Fig.1(a)) by the dotted line <sup>1</sup>. We see clearly that the CLEO data favour the colour-singlet process considered in this note.

Finally, we would like to emphasize that it is not difficult to check experimentally whether the colour-singlet process we consider indeed plays an important rôle for  $\Upsilon \rightarrow J/\Psi + X$ . This is because the  $J/\Psi$  in this colour-singlet process is definitely associated with open charm particles and its momentum is less than 3.3GeV/c.

In summary, both qualitative analyses and quantitative calculation show that the colour-singlet process  $\Upsilon \rightarrow J/\Psi + c\bar{c}g$  gives the branching ratio comparable to experimental data and a similar soft  $J/\Psi$  momentum spectrum as measured by CLEO Collaboration [8] as well. Among the processes which have been studied theoretically up to now, this is the only one which has such a feature. This seems to suggest that this process provide the main contribution to the inclusive  $J/\Psi$  production in  $\Upsilon$  decay.

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<sup>1</sup>We use Equation (24) in [6] to draw the line. We have changed the variable to  $J/\Psi$  momentum and normalized the expression on the right hand side of the equation to the width for this channel [6].



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## Figure Captions

Fig.1 Diagrams illustrating the colour-octet processes  $\Upsilon \rightarrow J/\Psi + gg$  [6] (a) and  $\Upsilon \rightarrow J/\Psi + g$  [7] (b).

Fig.2 One of the six diagrams for the direct colour-singlet process  $\Upsilon \rightarrow c\bar{c}(^3S_1, 1) + c\bar{c}g \rightarrow J/\Psi + c\bar{c}g$ . The real gluon vertex can be between those of the two virtual gluons or in either side of them. And the two virtual gluons can change their order.

Fig.3 The momentum spectrum of  $J/\Psi$  in the direct colour-singlet process we consider (solid line) and the momentum spectrum of  $\Psi(2S)$  multiplied by the branching ratio of  $\Psi(2S) \rightarrow J/\Psi + X'$  for the process  $\Upsilon \rightarrow \Psi(2S) + c\bar{c}g$  (dashed line). The CLEO data [8] is shown by dots. The result of the colour-octet process of Fig.1(a) is shown for comparison (dotted line).

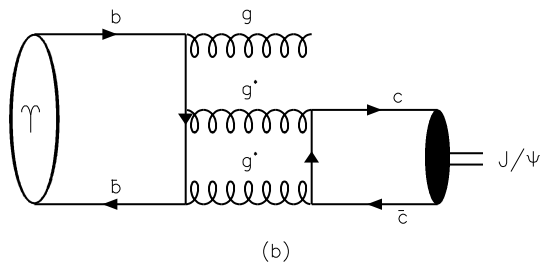
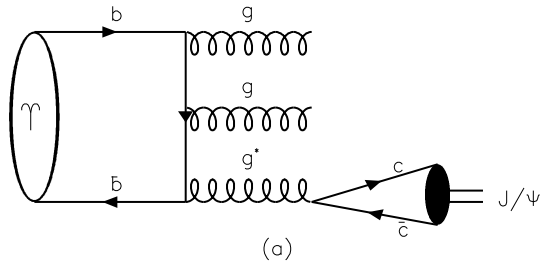


Figure 1:

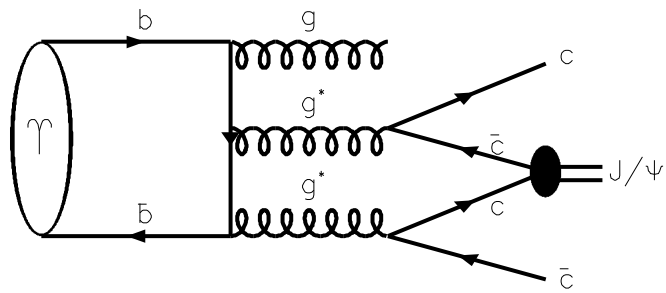


Figure 2:

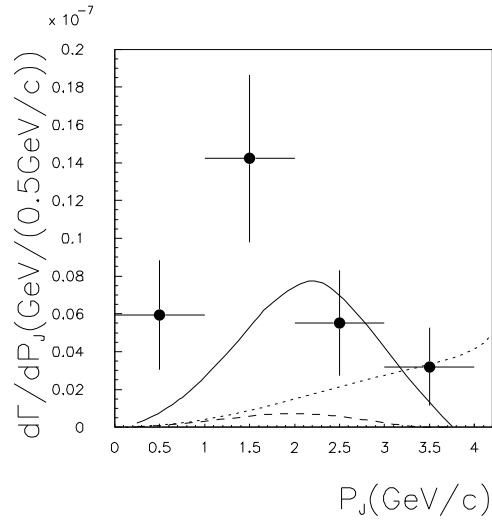


Figure 3: